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MODIFY BAYES ESTIMATION WITH EXTENSION LOOS FUNCTION FOR PARAMETER WEIBULL DISTRIBUTION

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ABSTRACT

The objective of this study is to introduce two estimators of parameter of Weibull distribution, which depends on modifying Bayes estimator and proposed Bayes estimator, depending on the extension loss function. Then, using a simulation study (MATLAB program), to find the best one based on MPE and MSE.

KEYWORDS: Bayes Estimation, Jeffery Prior Information, Simulation Study

INTRODUCTION

Elli and Rao (1986), estimated the shape and scale parameters of the Weibull distribution, by assuming a weighted squared error loss function. They minimized the corresponding expected loss, with respect to a given posterior distribution. Sinha & Sloan (1988), obtained Bayes estimator of three parameters, Weibull distribution compared to the posterior standard deviation estimates, with the corresponding asymptotic standard deviation of their maximum likelihood counterparts and numerical examples are given. In 2002, Klaus Felsenstein developed Bayesian procedures, for vague data. These data were assumed to be vague in the sense that, the likelihood is a mixture of the model distribution with error distribution. An extension of Jeffery prior information with square error and loss function in exponential distribution was studied by Al-Kutubi (2005).

In this paper, Alkutubi (2009) proposed an extension of Jeffery prior information, with a new loss function and then compared it with standard Bayes, for exponential distribution to find the best. In this paper, we will introduce two estimators to the parameters Weibull distribution, using modify Bayes estimation and extension loss function, We will then compare between them, by a simulation study to find the best one based on MPE and MSE.

Materials and Methods

Let $t_1, t_2, \dots, t_n$ be the life time of a random sample of size n with distribution function and probability density function.In the Weibull case, we assumed that the probability density function of the lifetime is given by

$$f(t,\theta,c) = \frac{c}{\theta}\left(\frac{t_i}{\theta}\right)^{c-1} \exp\left(\frac{-t}{\theta}\right)^{c}$$

The lifetime probability density function $f\,(t,\theta,c)$ are regarded as a conditional probability density function $f\,(t|\theta,c)$ where the marginal probability density function of θ is given by $g(\theta)$, the extension of Jeffery prior information.

$$f(t,\theta,c) = \frac{c}{\theta}\left(\frac{t}{\theta}\right)^{c-1} e^{\left(-\frac{t}{\theta}\right)^{c}}$$

$\frac{\partial \ln f(t,\theta,c)}{\partial \theta^2} = \frac{c}{\theta^2} - 2c\left(\frac{t}{\theta}\right)^{c-1}\left(\frac{t}{\theta^3}\right) - c(c-1)\left(\frac{t}{\theta}\right)^{c-2}\left(\frac{t}{\theta}\right)^2$=M

We find extension Jeffery prior by taking$g(\theta) \propto [I(\theta)]^{c_1}$, $c_1 \in R^+$

Where fisher information $I\ (\theta)$ is given by

$$I(\theta) = -n\ E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial \theta^2}\right)$$

$$E\left(\frac{\partial^2 \ln f(t,\theta,c)}{\partial \theta^2}\right) = E(M) = \int_0^\infty M\ \frac{c}{\theta}\left(\frac{t}{\theta}\right)^{c-1} e^{-\left(\frac{t}{\theta}\right)^c} dt = \frac{-c^2}{\theta^2}$$

Then $I(\theta) = \frac{nc^2}{\theta^2}$, so the extension of Jeffery prior information is given by

$$g(\theta) = k\frac{n^{c_1}}{\theta^{2c_1}}$$

The joint probability density function is:

$$H(t_1, \dots, t_n, \theta) = \prod_{i=1}^{n} f(t_i|\theta)g(\theta)$$

$$= \frac{kn^{c_1}}{\theta^{n+2c_1}} exp\left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right)$$

The marginal probability density function of $(t_i, \dots t_n, \theta, c)$ is given by:

$$P(t_1, \dots, t_n) = \frac{kn^{c_1}(n+2c_1-2)!}{\left(\sum_{i=1}^{n} t_i\right)^{n+2c_1-1}}$$

Then posterior distribution is,

$$= \frac{\theta^{-n-2c} exp\left(-\frac{\sum t_i}{\theta}\right)}{(\sum t_i)^{1-n-2c}(n+2c-2)!}$$

By using the squared error, loss function $\ell\left(\hat{\theta}-\theta\right)=c\left(\hat{\theta}-\theta\right)^2$, we can obtain the Risk function, such that

$$R(\hat{\theta},\theta) = EL(\hat{\theta},\theta) = \int_0^\infty c(\hat{\theta}-\theta)^2 \pi(\theta|t_1, \dots t_n)d\theta$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial \hat{\theta}} = 0$, then the Bayes estimator is$\widehat{\theta_1}$=$\frac{\sum t_i}{n+2c_{1-2}}$

By using the new extension loss function [1], we can get a new estimator of the parameter Weibull distribution, such that

$R(\hat{\theta},\theta) = \int_0^\infty \theta^c(\hat{\theta}-\theta)^2\ \pi(\theta|t_1, \dots t_n)d\theta$ Solving equation in above to get the second Bayes estimator

$$\hat{\theta}_2 = \frac{\sum t_i}{\left(\frac{2n-1}{c}\right)}$$

RESULTS

In a simulation study, we have chosen n=30, 60, 90 to represent small, moderate and large sample size, several values of parameter θ=0.3, 1, 1.2, and values of the new loss function c = 0.5,1.1. The number of replications used was R=1000. The simulation program was written by using the Matlab program. After the parameter was estimated, mean square error (MSE) and mean percentage error (MPE) were calculated to compare the methods of estimation, where

$$\text{MSE}\left(\hat{\theta}\right)=\frac{\sum_{i=1}^{1000}\left(\hat{\theta}-\theta\right)^2}{R} \text{ and } \text{MPE}\left(\hat{\theta}\right)=\frac{\left[\sum_{i=1}^{1000}\frac{\left|\hat{\theta}_i-\theta\right|}{\theta}\right]}{R}$$

The results of the simulation study are summarized and tabulated in Table 1 and Table 2, for the MSE and the MPE of the two estimators for all sample sizes and θ values, respectively. It is obvious from these tables that, modify Bayes estimator with the Jeffery prior information, $\widehat{\theta_1}$is the best estimator. But $\widehat{\theta_2}$ in most cases has the largest MSE and MPE

Table 1: The Ordering of the Estimators with Respect to MSE

Size	θ	C	Theta Hat	
			$\widehat{\theta}_1$	$\widehat{\theta}_2$
30	0.3	0.5	0.210	0.244
		1.1	0.224	0.225
	1	0.5	0.232	0.232
		1.1	0.235	0.238
	1.2	0.5	0.226	0.231
		1.1	0.223	0.235
60	0.3	0.5	0.125	0.131
		1.1	0.121	0.131
	1	0.5	0.121	0.120
		1.1	0.122	0.123
	1.2	0.5	0.122	0.123
		1.1	0.121	0.122
90	0.3	0.5	0.091	0.091
		1.1	0.091	0.091
	1	0.5	0.081	0.085
		1.1	0.080	0.085
	1.2	0.5	0.081	0.081
		1.1	0.081	0.084

Table 2: The Ordering of the Estimators with Respect to MPE

Size	θ	C	Theta Hat	
			$\widehat{\theta}_1$	$\widehat{\theta}_2$
30	0.3	0.5	0.210	0.214
		1.1	0.210	0.214
	1	0.5	0.211	0.223
		1.1	0.211	0.223
	1.2	0.5	0.212	0.212
		1.1	0.212	0.211
60	0.3	0.5	0.113	0.114
		1.1	0.113	0.114
	1	0.5	0.112	0.113
		1.1	0.112	0.113
	1.2	0.5	0.111	0.112
		1.1	0.111	0.112
90	0.3	0.5	0.004	0.004
		1.1	0.004	0.004
	1	0.5	0.004	0.005
		1.1	0.004	0.005
	1.2	0.5	0.005	0.006
		1.1	0.005	0.006

CONCLUSIONS

The modify Bayes estimator with Jeffery prior information $\hat{\theta}_1$ is the best estimator, when compared it with the other estimators. We can easily conclude that, MSE and MPE of Bayes estimators decrease with an increased of sample size.

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